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PHYSICS

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DEPENDENCE OF REACTIVITY OF THE WATER BOILER ON THE MASS OF 25* IN THE SPHERE

By J. Hinton

ABSTRACT

A semitheoretical determination of the percentage change in reactivity of the water boiler per gram of 25 removed from the sphere is presented. The method consists in calculating the external neutron source required to keep level the neutron intensity in the boiler when x grams of 25 are removed from the sphere. Use is made of the experimentally determined neutron-distribution curves. It is found that 18.2 g of 25 subtracted from the sphere produces 1 per cent change in K.

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The evaluation of many experimental results obtained with the water boiler requires a knowledge of the change in reproductive factor per gram of 25 removed from the sphere.

Several experimental methods of determining the dependence of reactivity on grams of 25 have been suggested by certain members of the theoretical department in order to avoid the trouble of calculating the dependence theoretically. One of these, the boron-bubble method, was carried out by Kerst and de Hoffman. It seemed worth while as a check on this method to carry out also a semitheoretical determination. In the calculation here presented, use is made of the experimentally observed neutron-distribution curves. This makes the procedure much simpler and perhaps somewhat more reliable than a complete theoretical determination.

If K is defined as the number of fissions produced per generation by each fission in the boiler, then if K equals 1, each fission produces on the average one more fission, and the boiler runs at constant power.

When the boiler is set to run with K equals 1, the quantity to be determined is the change in K if x grams of 25 is removed from the sphere.

If for a moment the spacial distribution of the activity is disregarded, the boiler may be run at a constant rate even when K is less than 1 by adding a source. If this source is equivalent to ϕ fissions per unit time, then the total number of fissions F will be given by

$$F = \phi/(1 - K)$$

or

$$\delta K = 1 - K = \phi/F \tag{1}$$

This formula can be used for calculating the change in K due to removal of x grams of 25 from the sphere. For this formula, first it must be determined what source must be added in order to keep the intensity level when x grams is removed so that the system becomes undercritical. Since both fissions F and source ϕ are actually functions of r, proper averages must be taken in calculating δK.

*The number 25 is used throughout this report as an abbreviation for U^{235}.

In order to determine the added source necessary when x grams is removed, assume that the boiler is critical with m grams of 25 in the sphere; then the number of fissions per cubic centimeters would be

$$F_1(r) = n(r)\ v(m/V)\ \sigma_f \tag{2}$$

where σ_f is the fission cross section per gram, and V is the volume of the sphere.

If x grams is removed, leaving the volume unchanged, the neutron flux as well as the density of 25 will change. Assuming we start with the same number of fissions in the first generation, we shall then get in the second generation.

$$F_2(r) = [n(r) - \epsilon(r)]\,[(m - x)/V]\ v\sigma_f = F_1(r) - \phi(r) \tag{3}$$

where $\epsilon(r)$ represents a small variation in neutron density.

In order to keep the intensity unchanged, namely, to have $F_2 = F_1$, an external source has to be added, distributed in such a way as to produce the fission distribution $\phi(r)$. If F(r) and $\phi(r)$ were everywhere proportional, then δK would simply be equal to $\phi(r)/F(r)$. Actually it results that ϕ and F have a very different dependence on r. Nevertheless, a formula similar to Eq. 1 can be applied as follows.

A source of fission neutrons is put in a boiler that is almost critical. The boiler acquires a steady intensity that depends on the position at which the source is put and which has a maximum when the source is at the center. Now defined is a function $\theta(r)$ equal to the ratio of the intensity when the source is placed at a distance r from the center to the intensity when the same source is placed at the center.

Thus by multiplying any source placed at a position r by the function $\theta(r)$, the source is determined which it would be necessary to place in the center in order to make the boiler run at the same intensity.

For a source of Ra-Be neutrons, the function $\theta(r)$ was determined experimentally by Kerst. His results are plotted in Fig. 1. It is to be expected that the function $\theta(r)$ is somewhat dependent on energy of the neutrons emitted by the source and is flatter for neutrons of high energy. For fission neutrons having a lower average energy than the Ra-Be neutrons, a somewhat steeper curve is expected. On the other hand, it is expected that the fission-neutron curve would be slightly flatter than a thermal-neutron curve. The desired curve would therefore lie somewhere between the Ra-Be curve and a thermal-source curve of the function $\theta(r)$. It can be shown that the thermal-distribution curve is very nearly equivalent to a thermal-source curve. It is exactly equivalent with an infinite water tamper.

The calculation has been carried out both by taking $\theta(r) = \theta(r)_{Ra\text{-}Be}$ and $\theta(r) = n(r)$, where n(r) is the experimental thermal-distribution curve. The result is practically independent of the choice.

As stated above, a source placed anywhere can be reduced to an equivalent source placed at the center by multiplying it by $\theta(r)$. Consequently, the source distribution $\phi(r)$ will be equivalent to a concentrated source of strength $4\pi \int_0^R \phi(r) r^2 \theta(r) dr$ at the center. And in a similar way, the source distribution F(r) is equivalent to $4\pi \int_0^R F(r) r^2 \theta(r) dr$. Equation 1 can now be used in the form

$$\delta K = \frac{\int_0^R \phi(r) r^2 \theta(r) dr}{\int_0^R F(r) r^2 \theta(r) dr} \tag{4}$$

The fission-source distribution F(r) is assumed to be proportional to the thermal-neutron distribution n(r) according to Eq. 2. It is therefore obtained from the experimental thermal curve (Fig. 1).

The remaining part of the calculation consists in determining the function $\phi(r)$. Since according to Eq. 3, this depends on $\epsilon(r)$, the change in neutron density as a function of radius, $\epsilon(r)$ must be determined first.

An equation in $\epsilon(r)$ may be obtained by taking the difference of the diffusion equations with m and (m-x) grams of 25 in the sphere.

It is assumed that the concentration of hydrogen is practically unchanged. Therefore the scattering mean free path and the slowing-down process, both of which depend mainly on hydrogen, are assumed not to change when x grams are subtracted.

Since only the absorption mean free path and the neutron density change, the following equations may be written, where index s refers to "sphere," and index t refers to "tamper"

With m grams	$n = n(r)$	$1/\Lambda_s \quad 1/\Lambda_s$
With m-x grams	$n = n(r) + \epsilon(r)$	$1/\Lambda_s \rightarrow 1/\Lambda_s - \eta$

	Sphere	Boundary	Tamper	
With m grams	$(\lambda_s v/3)\Delta n - nv/\Lambda_s + q_s = 0$		$(\lambda_t v/3)\Delta_n - nv/\Lambda_t + q_t = 0$	(5)
With m-x grams	$(\lambda_s v/3)\Delta(n-\epsilon)-(n-\epsilon)(1/\Lambda_s-\eta) + q_s = 0$	$\Vert$	$(\lambda_t v/3)\Delta(n-\epsilon) - (n-\epsilon)v/\Lambda_t + q_t = 0$	(6)
		($\frac{1}{32}$-in. stainless-steel sphere wall)		

With the boundary condition that the number of neutrons diffusing through the stainless-steel wall from the sphere is equal to the number diffusing back into the sphere from the tamper plus the number absorbed in the wall

$$(\lambda_s v/3)\, n'_{R-} - (\lambda_t v/3)\, n'_{R+} + \sigma_w n_R v = 0$$

where σ_w is the absorption cross section per square centimeter of the wall (0.015 cm^2/cm^2 for a $\frac{1}{32}$-in. steel wall). Subtracting Eq. 6 from Eq. 5 the equation in (r) is

$$\Delta\epsilon(r) - \left[\epsilon(r)/L_s^2\right] = n(r)\,\eta \qquad \Vert \qquad \Delta\epsilon(r) - \left[\epsilon(r)/L_t^2\right] = 0$$

where $L^2 = \Lambda\lambda/3$. The boundary condition for $\epsilon(r)$ is the same as that for n(r).

Substituting $U(r) = \epsilon(r)r\ \lambda/3\eta$, the equations to be solved are

$$U''(r) - \frac{U(r)}{L_s^2} = rn(r) \tag{7}$$

$$U''(r) - \frac{U(r)}{L_t^2} = 0 \tag{8}$$

$$U'(R+) = \frac{\lambda_s}{\lambda_t} U'(R-) + U(R)\left[\frac{3\sigma_w}{\lambda_t} + \frac{1-\lambda_s/\lambda_t}{R}\right] \tag{9}$$

$$U(O) = 0 \tag{10}$$

Taking $L_t = 30$ cm, $\rho_t = 2.69$, $\lambda_t/3 = .5$, $\Lambda_t = 1800$, $1/\Lambda_s = 0.075 + 10^{-4}x$, and $\lambda_s/3 = 0.153$, the solution is

$$U(r) = \frac{e^{ar}}{2a}\int_0^r rn(r)e^{-ar}\,dr - \frac{e^{-ar}}{2a}\int_0^r rn(r)e^{+ar}\,dr - 1.4914(e^{ar} - e^{-ar}) \tag{11}$$

where $r < R$, and $a = 1/L_s$. The integrals in this expression are evaluated numerically, taking the experimental Mn-distribution curves as n(r). U(r) being known, $\phi(r)$ is calculated by Eq. 3. Then δK is obtained by numerical integration of Eq. 4.

By this method it was found that 18.2 g subtracted from the sphere produces 1 per cent change in K. The result of the boron-bubble experiment, as given in LADC-816, is in exact agreement with this calculation.

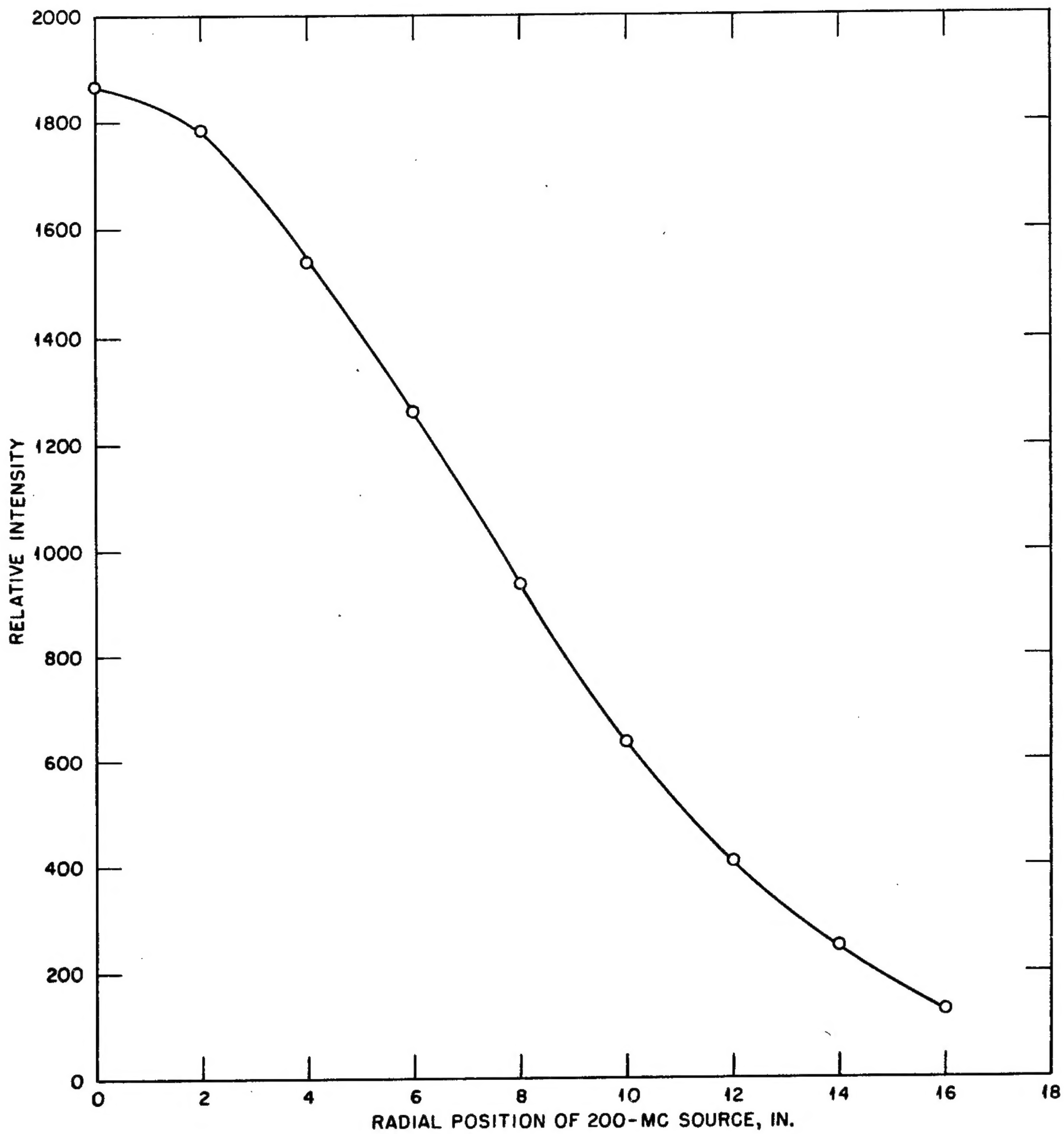

Fig. 1—Experimental thermal curve.

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